

SPRING 2025 MATH 590: QUIZ 4

Name:

1. Let $A = (a_{ij})$ be an $n \times n$ matrix. Write the formula for the expansion of $|A|$ along the second column. (3 points)

Solution. $|A| = \sum_{i=1}^n (-1)^{i+2} a_{i2} |A_{i2}|.$

2. Suppose $A = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$. Use expansion along a row or column to calculate $|A|$. (3 points)

Solution. Expanding along the third row, we have:

$$|A| = 0 \cdot \begin{vmatrix} b & c \\ d & e \end{vmatrix} - 0 \cdot \begin{vmatrix} a & c \\ 0 & e \end{vmatrix} + f \cdot \begin{vmatrix} a & b \\ 0 & d \end{vmatrix} = 0 - 0 + fad = fad.$$

3. Use Cramer's rule to solve for $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. (4 points)

Solution. $x = \frac{\begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}} = -\frac{2}{5}$ and $y = \frac{\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}} = \frac{4}{5}.$